# Practical Class 9

# Test Paper No. 1

## "Differential Calculus of Functions of Several Variables"

**30 Variants**

Time: 90 minutes

## VARIANT 1

**1.** Find the domain of the function:
$z = \sqrt{9 - x^2 - y^2} + \ln(xy)$

**2.** Find the partial derivatives and the total differential:
$z = x^2 e^y + \operatorname{arctg}(xy)$

**3.** Find the derivative of the composite function:
$z = \ln(u^2 + v)$, $u = \sin x$, $v = \cos x$; $\dfrac{dz}{dx}$

**4.** Investigate for local extremum:
$z = x^3 + 3xy^2 - 15x - 12y$

## VARIANT 2

**1.** Find the domain of the function:
$z = \sqrt{x - y^2} + \dfrac{1}{\sqrt{4 - x^2 - y^2}}$

**2.** Find the partial derivatives and the total differential:
$z = \sin(x^2 y) + \ln(x + y)$

**3.** Find the derivative of the composite function:
$z = x^y$, $x = \cos t$, $y = \sin t$; $\dfrac{dz}{dt}$

**4.** Investigate for local extremum:
$z = x^3 + y^3 - 3xy$

## VARIANT 3

**1.** Find the domain of the function:
$z = \sqrt{1 - x^2} + \sqrt{4 - y^2}$

**2.** Find the partial derivatives and the total differential:
$z = \dfrac{x}{y^2} + \operatorname{arctg} \dfrac{y}{x}$

**3.** Find the derivative of the composite function:
$z = \operatorname{arctg}(xy)$, $x = e^t$, $y = e^{2t}$; $\dfrac{dz}{dt}$

**4.** Investigate for local extremum:
$z = 2x^2 + y^2 - xy - 7x$

## VARIANT 4

**1.** Find the domain of the function:
$z = \ln(4 - x^2 - 4y^2)$

**2.** Find the partial derivatives and the total differential:
$z = \dfrac{\cos(xy)}{x} + e^{x^2 y}$

**3.** Find the derivative of the composite function:
$z = \arcsin(xy)$, $x = t^2$, $y = 1/t$; $\dfrac{dz}{dt}$

**4.** Investigate for local extremum:
$z = xy + \dfrac{50}{x} + \dfrac{20}{y}$

## VARIANT 5

**1.** Find the domain of the function:
$z = \dfrac{1}{\sqrt{x^2 + y^2 - 1}} + \sqrt{9 - x^2 - y^2}$

**2.** Find the partial derivatives and the total differential:
$z = x^y + \ln \dfrac{x}{y}$

**3.** Find the derivative of the composite function:
$z = \ln(e^x + e^y)$, $y = x^2$; $\dfrac{dz}{dx}$

**4.** Investigate for local extremum:
$z = x^2 + xy + y^2 - 2x - 3y$

## VARIANT 6

**1.** Find the domain of the function:
$z = \sqrt{x^2 + y^2 - 4} + \arcsin \dfrac{x}{3}$

**2.** Find the partial derivatives and the total differential:
$z = \operatorname{tg}(x^2 + y^2) + \dfrac{x}{y}$

**3.** Find the derivative of the composite function:
$z = u^v$, $u = \sin x$, $v = \cos x$; $\dfrac{dz}{dx}$

**4.** Investigate for local extremum:
$z = 2x^3 - xy^2 + 5x^2 + y^2$

## VARIANT 7

**1.** Find the domain of the function:
$z = \sqrt{y - x^2} + \dfrac{1}{\sqrt{1 - x^2 - y^2}}$

**2.** Find the partial derivatives and the total differential:
$z = \dfrac{x}{\sqrt{x^2 + y^2}} + \ln(1 + xy)$

**3.** Find the derivatives of the implicit function:
$x^2 + y^2 + z^2 = 4z$; $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$

**4.** Investigate for local extremum:
$z = x^2 + y^2 - \ln x - \ln y$

## VARIANT 8

**1.** Find the domain of the function:
$z = \ln(xy - 1) + \sqrt{4 - x^2 - y^2}$

**2.** Find the partial derivatives and the total differential:
$z = \arcsin \dfrac{x}{y} + e^{x/y}$

**3.** Find the partial derivatives of the composite function:
$z = \operatorname{arctg} \dfrac{u}{v}$, $u = x + y$, $v = x - y$; $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$

**4.** Investigate for local extremum:
$z = x^3 + 8y^3 - 6xy + 1$

## VARIANT 9

**1.** Find the domain of the function:
$z = \dfrac{1}{\sqrt{x^2 - y}} + \sqrt{9 - x^2 - y^2}$

**2.** Find the partial derivatives and the total differential:
$z = \ln(x + \ln y) + \dfrac{y}{x}$

**3.** Find the partial derivatives of the composite function:
$z = x^2 y$, $x = u^2 - v^2$, $y = 2uv$; $\dfrac{\partial z}{\partial u}$, $\dfrac{\partial z}{\partial v}$

**4.** Investigate for local extremum:
$z = xy + \dfrac{1}{x} + \dfrac{1}{y}$

## VARIANT 10

**1.** Find the domain of the function:
$z = \sqrt{4x - y^2} + \ln(1 - x^2 - y^2)$

**2.** Find the partial derivatives and the total differential:
$z = \dfrac{\sin(xy)}{x} + \sqrt{x^2 + y^2}$

**3.** Find the derivatives of the implicit function:
$e^z = x^2 + y^2 + z$; $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$

**4.** Investigate for local extremum:
$z = 2x^2 + 2xy - y^2 - 4x + 6y$

## VARIANT 11

**1.** Find the domain of the function:
$z = \sqrt{\dfrac{x}{y} - 1} + \sqrt{9 - x^2 - y^2}$

**2.** Find the partial derivatives and the total differential:
$z = \operatorname{arctg} \dfrac{x}{y} + \ln \sqrt{x^2 + y^2}$

**3.** Find the partial derivatives of the composite function:
$z = f(x^2 + y^2, xy)$; $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$

**4.** Investigate for local extremum:
$z = x^2 + xy + y^2 - 4\ln x - 10\ln y$

## VARIANT 12

**1.** Find the domain of the function:
$z = \sqrt{x^2 - y} + \sqrt{y + 2}$

**2.** Find the partial derivatives and the total differential:
$z = \dfrac{x^2 - y^2}{x^2 + y^2} + \ln(1 + x^2 y^2)$

**3.** Find the partial derivatives of the composite function:
$z = \ln(x^2 + y^2)$, $x = u\cos v$, $y = u\sin v$; $\dfrac{\partial z}{\partial u}$, $\dfrac{\partial z}{\partial v}$

**4.** Investigate for local extremum:
$z = 3x^2 - x^3 + 3y^2 + 4y$