Practical Class 8. Review for Test No. 1

Course "Mathematical Analysis. Part 2".
Higher Mathematics.

Objective of the Class

Systematize knowledge on the topic "Differential Calculus of Functions of Several Variables", review key types of problems, and prepare for the test paper.

Organizational Moment (5 minutes)

- Information about the structure of the test paper (30 variants, 4 problems in each)
- Duration: 90 minutes
- Rules: no notes, no calculators, only writing utensils
- Grading criteria

Review of Theoretical Material (20 minutes)

1. Domain of a Function of Several Variables

Key Points

- The domain of a function $z = f(x, y)$ is the set of points (x, y) for which the expression makes sense.
- Typical restrictions:
  - Denominator $\neq 0$
  - Expression under square root ≥ 0
  - Argument of logarithm > 0
  - Argument of arcsine/arccosine $\in [-1, 1]$
- The answer can be described using inequalities and sketched on the plane.

2. Partial Derivatives and Total Differential

Key Points

- $\frac{\partial z}{\partial x}$ is the derivative with respect to x assuming y constant.
- $\frac{\partial z}{\partial y}$ is the derivative with respect to y assuming x constant.
- Total differential: $dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$.
- For composite functions $z = z(u, v)$, $u = u(x)$, $v = v(x)$:

$$\frac{dz}{dx} = \frac{\partial z}{\partial u}\frac{du}{dx} + \frac{\partial z}{\partial v}\frac{dv}{dx}$$

- For composite functions $z = z(u, v)$, $u = u(x, y)$, $v = v(x, y)$:

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x}$$

3. Implicit Functions

Key Points

If the function $z = z(x, y)$ is defined implicitly by the equation $F(x, y, z) = 0$, then:

$$\frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z}, \quad \frac{\partial z}{\partial y} = -\frac{F'_y}{F'_z} \quad (F'_z \neq 0)$$

4. Local Extremum of a Function of Two Variables

Algorithm

1. Find stationary points by solving $\begin{cases} f'_x = 0 \\ f'_y = 0 \end{cases}$.
2. Compute $A = f''_{xx}$, $B = f''_{xy}$, $C = f''_{yy}$ at each point.
3. Compute $D = AC - B^2$:
   - If $D > 0$ and $A > 0$ — minimum
   - If $D > 0$ and $A < 0$ — maximum
   - If $D < 0$ — saddle point
   - If $D = 0$ — additional investigation needed

Typical Problems (50 minutes)

1. Domain of a Function of Several Variables

Key Points

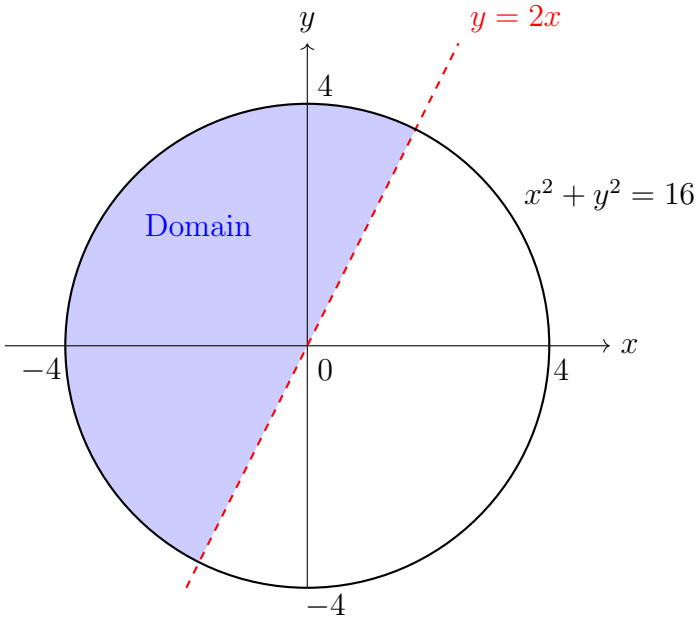
- The domain of a function $z = f(x, y)$ is the set of points (x, y) for which the expression makes sense.
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  - Denominator $\neq 0$
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Example 1 (Disk + Half-plane)

Find the domain of the function $z = \sqrt{16 - x^2 - y^2} + \ln(y - 2x)$.

Solution:

1. For the square root: $16 - x^2 - y^2 \geq 0 \Rightarrow x^2 + y^2 \leq 16$ (a disk of radius 4 centered at the origin).
2. For the logarithm: $y - 2x > 0 \Rightarrow y > 2x$ (the region above the line $y = 2x$).



Answer: $D = \{(x, y) \mid x^2 + y^2 \leq 16, y > 2x\}$ — the part of the disk lying above the line $y = 2x$.