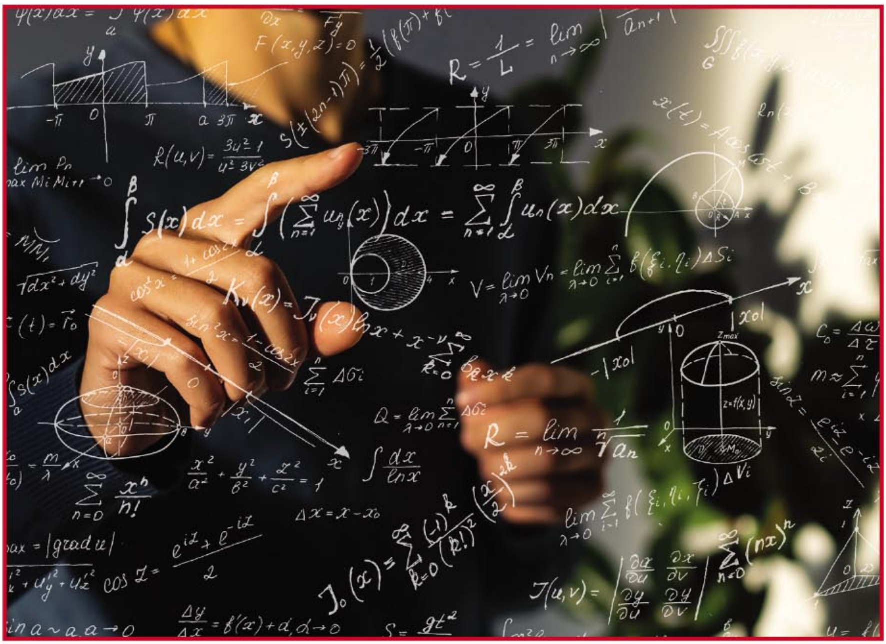
УЧЕБНОЕ ПОСОБИЕ

ФИНАНСОВЫЙ УНИВЕРСИТЕТ
ПРИ ПРАВИТЕЛЬСТВЕ РОССИЙСКОЙ ФЕДЕРАЦИИ

В.Б. Гисин

МАТЕМАТИКА

МАТЕМАТИЧЕСКИЙ АНАЛИЗ 1



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Математика. Математический анализ 1: учебное пособие для студентов, обучающихся по направлению «Экономика» 38.03.01 — программа подготовки бакалавра

Пособие предназначено для студентов, изучающих математику на английском языке. Пособие содержит учебный материал, относящийся к введению в математический анализ и таким его разделам, как дифференциальное и интегральное исчисление функций одной переменной.

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PRECALCULUS

1.1. BASIC NOTATIONS

Signs and Symbols

$N = \{1, 2, \ldots\}$	set of natural numbers
$Z = \{0, \pm 1, \pm 2, \ldots\}$	set of integer numbers
$Q = \{\frac{a}{b} \mid a, b \in Z, b \neq 0\}$	set of rational numbers
R	set of real numbers
$C = \{a + bi \mid a, b \in R, i^2 = -1\}$	set of complex numbers
$a = b$	a equals b, a is equal to b, identity
$a \neq b$	a is not equal to b, a does not equal to b, a is other than b
$a < b$	a is less than b
$a \leq b$	a is less than or equal to b
$a > b$	a is greater than b
$a \geq b$	a is greater than or equal to b
$a > 0$	a is a positive number, a is greater than 0
$a \geq 0$	a is a non-negative number

$a+b$	addition, *a* plus *b*, sum of *a* and *b*, *a*, *b* are the summands
$a-b$	subtraction, *a* minus *b*, difference between *a* and *b*
$a \cdot b$	multiplication, *a* times *b*, product of *a* and *b*, *a*, *b* are the factors
$\frac{a}{b}$	division, *a* divided by *b*, *a* over *b*, quotient of *a* and *b*, *a* is the numerator, *b* is the denominator
½, 2/3, 2½	one half, two thirds, two and a half
0.01	nought point nought one
5%, 2/5%	five per cent, two fifth per cent
$\sum_{i=1}^{n} a_i = a_1 + \ldots + a_n$	sum over a_i of *i* equals 1 up to *n*
$\prod_{i=1}^{n} a_i = a_1 \cdot \ldots \cdot a_n$	product over a_i of *i* equals 1 up to *n*
∞	infinity

Functions

$f: X \to Y$	*f* is a transformation of *X* into *Y*
X, $D(f) = f^{-1}(Y)$	domain of *f*
Y, $R(f) = f(X)$	codomain of *f*, range of *f*
x^n	*x* to the power of *n*, *n*th power of *x* for $n \geq 0$
x^2	*x* squared

x^3	x cubed
$\sqrt{x}$, $\sqrt[n]{x}$	square root of x, n-th root of x
$n!$	n factorial
$\lvert x \rvert$	absolute value of x
$\operatorname{sgn} x$	sign of x ($\operatorname{sgn} 5 = 1$, $\operatorname{sgn} -3 = -1$, $\operatorname{sgn} 0 = 0$)
e^x , $\exp x$	exponential function of x, e to the power of x
$\log_a x$	logarithm (log) of x (to) base a
$\ln x$	natural logarithm (log) of x (to base e)
$\sin x$	sine of x
$\cos x$	cosine of x
$\sec x = \frac{1}{\cos x}$	secant of x
$\csc x = \frac{1}{\sin x}$	cosecant of x
$\tan x$, $\operatorname{tg} x$	tangent of x
$\cot x$, $\operatorname{ctg} x$	cotangent of x
$\arcsin x$, $\sin^{-1} x$	arc sine of x, inverse sine of x
$\arccos x$, $\cos^{-1} x$	arc cosine of x, inverse cosine of x
$\arctan x$, $\operatorname{arctg} x$	arc tangent of x, inverse tangent of x

Sets

$\{a, b, c, ...\}$	set with the elements a, b, c, ...
$a \in A$	a is an element of A, e.g. $3 \in Z$ (3 is an integer)

$a \notin A$	a is an element of A, e.g. $\sqrt{2} \notin Q$ ($\sqrt{2}$ is not rational number)
$\{x \in M \| P(x)\}$	The set of all elements in M satisfying condition P, e.g. $\{x \in Z \| 2 < x < 5\} = \{3,4\}$
$A \subseteq B$	A is a subset of B (any element of A is an element of B), A is included in B
$A \subset B$	A is a proper subset of B (is a subset and unequal)
$A \cup B$	A union B, A or B, includes all occurring elements (x is in $A \cup B$ if and only if x is in A or x is in B)
$A \cap B$	A intersection B, A and B, includes all common elements (x is in $A \cup B$ if and only if x is in A and x is in B)
$A \setminus B$	A not B include all elements of A that are not in B
$A \times B$	A cross B, cartesian product of A and B, set of all (ordered) pairs from A and B
$\underbrace{A \times \ldots \times A}_{n} = A^n$	the set of all ordered n-tuples $(x_1, \ldots, x_n)$, $x_i \in A$, $i = 1, \ldots, n$, e.g. R^2, R^3
$\varnothing$	Empty set

Greek Alphabet

Letters (upper case, lower case)	Name	Pronunciation
Α α	alpha	′ælfə
Β β	beta	′bi:tə
Γ γ	gamma	′gæmə
Δ δ	delta	′deltə

Ε ε	epsilon	′epsəlɔn
Ξ ξ	xi	ksai
Η η	eta	′i:tə
Ζ ζ	zeta	′zi:tə
Ι ι	iota	ai′əʊtə
Κ κ	kappa	′kæpə
Λ λ	lambda	′læmbə
Μ μ	mu	′mju:
Ν ν	nu	′nju:
Ο ο	omicron	′əʊməkrʊn
Π π	pi	pai
Ρ ρ	rho	rəʊ
Σ σ	sigma	′sigmə
Τ τ	tau	taʊ
ϒ υ	upsilon	′ʌpsəlʊn
Θ θ	theta	′θi:tə
Φ φ	phi	fai
Ψ ψ	psi	psai
Χ χ	chi	kai
Ω ω	omega	əʊ′migə

1.2. GRAPHS ON THE CARTESIAN PLANE

Rectangular coordinates

Ordered pairs of real numbers can be represented by points in a plane called the ***rectangular coordinate system***, or the ***Cartesian plane*** (Figure 1).

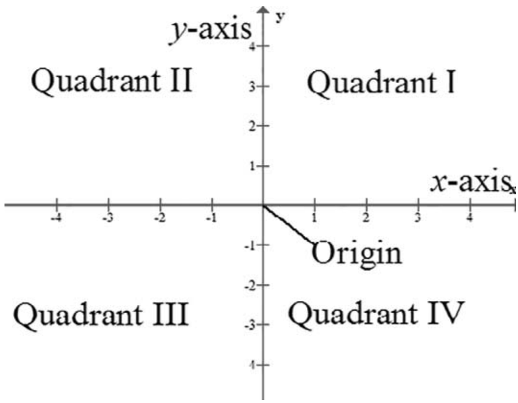


Figure 1. Coordinate system

The horizontal real number line is usually called the ***x-axis***, and the vertical real number line is usually called the ***y-axis***. The point of intersection of these two axes is the origin, and the two axes divide the plane into four ***quadrants***.

The scatter plot is a way to represent the data graphically. To sketch a scatter plot of the data presented by a finite number of pairs (x, y), plot the points (x, y).

Pythagorean Theorem and the Distance Formula

For a right triangle with hypotenuse length c and sides lengths a and b, we have

$$a^2 + b^2 = c^2 .$$

The converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is **(Fig 2)**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

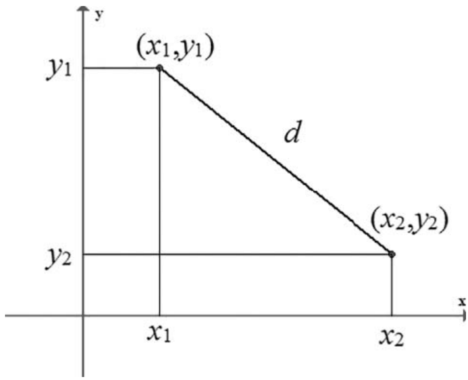


Figure 2. D2-distance

The Midpoint Formula

The midpoint M of the line segment joining the points (x_1, y_1) and (x_2, y_2) is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Example. The production of Lada-Vesta cars in January 2021 amounted to 8879, and in March — to 13349. Without knowing any additional information, what would be estimated the February 2021 production? (Source: AvtoVAZ OJSC).

Let (n, Q_n) be production in month n of year 2021. Assuming that production followed a linear pattern, we suppose that $(2, Q_2)$ is the midpoint of the line segment joining the points $(1, Q_1)$ and $(3, Q_3)$. So

$$(2, Q_2) = \left(\frac{1+3}{2}, \frac{8879+13349}{2}\right) = (2, 11114).$$

Thus, the February production is estimated at 11114. The actual amount is 11298. The error is about 1.6%.

To shift a point a units to the right, add a to the x-coordinate. To shift a point b units up, add b to the y-coordinate: if (x, y) is an original point then the translated point is $(x+a, y+b)$.

The Graph of an Equation

Let $F(x, y) = 0$ be an equation relating x and y. The graph of this equation is the set of the solution points, i.e. the points (x, y) satisfying $F(x, y) = 0$. In particularly, the graph of a function $y = f(x)$ is the set of points (x, y) such that $y = f(x)$.

Solution points of an equation that have zero as either the x-coordinate or the y-coordinate are called *intercepts*.

To find x-intercepts, we have to solve the equation $F(x, 0) = 0$. To find y-intercepts, we have to solve the equation $F(0, y) = 0$. If $y = f(x)$ and $f(x)$ is defined at $x = 0$ then the y-intercept is $(0, f(0))$. To find x-intercepts, we have to solve the equation $f(x) = 0$.

A graph is symmetric with respect to the x-axis if, whenever (x, y) is on the graph, $(x, -y)$ is also on the graph (replacing y with $-y$ yields an equivalent equation).

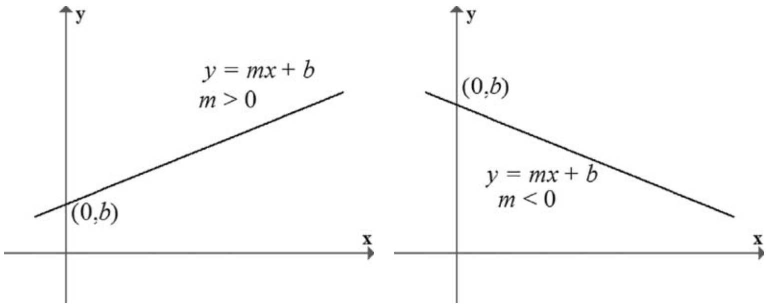
A graph is symmetric with respect to the y-axis if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph (replacing x with $-x$ yields an equivalent equation).

A graph is symmetric with respect to the origin if, whenever (x, y) is on the graph, $(-x, -y)$ is also on the graph (replacing x with $-x$ and y with $-y$ yields an equivalent equation).

The graph of a linear equation

$$y = mx + b$$

is a line whose ***slope*** is m and whose y-intercept is (0, b) (Figure 3).



a) Positive slope, line rises b) Negative slope, line falls

Figure 3. Lines

A vertical line has an equation of the form $x = a$. The slope of a vertical line is undefined.

The ratio

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

represents the slope of the line that passes through the points (x_1, y_1) and (x_2, y_2). The equation of the line with slope m passing through the point (x_1, y_1) may be written as follows

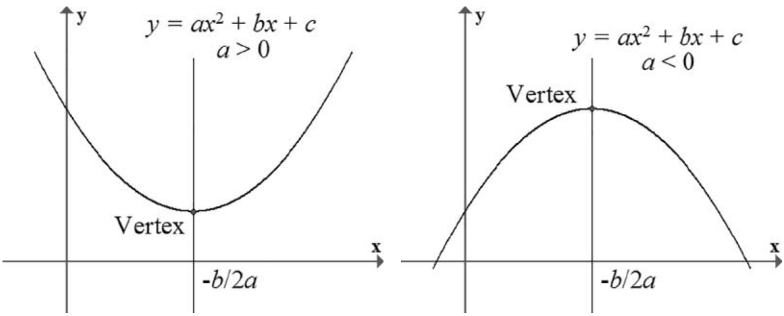
$$y - y_1 = m(x - x_1)$$

(***point-slope form***).

The graph of a quadratic equation

$$y = ax^2 + bx + c$$

with $a \neq 0$ is ***parabola***. Parabola is a U-shaped curve. Any parabola is symmetric with respect to its axis. The point where the axis intersects the parabola is the *vertex* of the parabola. When the leading coefficient *a* is positive, the parabola opens upward. When *a* is negative, the parabola opens downwards (Figure 4).



(a)	(b)
Leading coefficient is positive	Leading coefficient is negative
Vertex is the lowest point	Vertex is the highest point
Parabola opens upward	Parabola opens downward

Figure 4. Parabolas

The graph of equation

$$(x - a)^2 + (y - b)^2 = r^2$$

is a ***circle*** with its center at (a,b) and radius *r*.

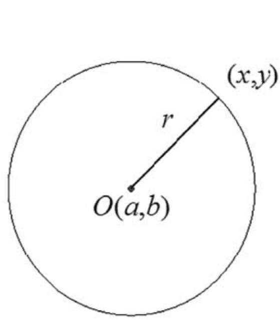


Figure 5. A circle with its center at (a,b) and radius r

1.3. FUNCTIONS

Basic Notions

A function *f* from a set X to a set Y assigns to each element $x \in X$ exactly one element $y \in Y$. The set X is the ***domain*** (or set of inputs) of the function *f*, and the set Y contains the ***range*** (or set of outputs). The domain of *f* is denoted by $D(f)$ or D_f and the range by $R(f)$ or R_f.

If A is a subset of $D(f)$ then $f(A)$ is the set of all y such that $y = f(x)$ for some $x \in A$. Obviously $f(D_f) = R_f$.

The ***composition*** of the function *f* with the function *g* is defined for x and assigns $f(g(x))$ to x if $g(x) \in D_f$. The function *f* is the outer function; the function *g* is the inner function.

Example. Let $f(x) = x^2$ and $g(x) = x + 1$. Then $f(g(x)) = (x+1)^2$ considering $f(x) = x^2$ as the outer function, and $g(x) = x + 1$ as the inner function. Similarly, considering $g(x) = x + 1$ as the outer function, and $f(x) = x^2$ as the inner function we get $g(f(x)) = x^2 + 1$.

Let *f* and *g* be functions. The function *g* is the ***inverse*** function of the function *f* if the following conditions hold: $f(g(x)) = x$ for every $x \in D_g$; $g(f(x)) = x$ for every $x \in D_f$.

The function g is denoted by f^{-1} (read "*f*-inverse"). By the definition

$$f(f^{-1}(x)) = x \text{ and } f^{-1}(f(x)) = x.$$

If *g* is the inverse function of *f*, then it must also be true that *f* is the inverse function of *g*. So, the functions *f* and *g* are inverse functions of each other.

A function *f* has an inverse function if and only if *f* is one-to-one, i.e. each value of y dependent variable corresponds to exactly one value of the independent variable. In other words, equation $f(x) = y$ has the only solution for every *y* from the range of *f*.